**SC531 – Lecture #01**

Why do we study this subject? How is it related to Computer Science? What is the famous “Turing test"?

VERY BROADLY, we study this subject because we would like to solve practical problems – with or without computers – through correct “reasoning".

But what is the meaning of “reasoning”?

Remember that *Homo Sapiens* have had this quality for hundreds of thousands of years. The only way one can understand the process which we call “reasoning” is by studying the range of human activities.

So, in that spirit, let us get started.

Quick review of Propositional Logic

**Logical deduction**[understood as a form of "**inference**"]

Logic involves making deductions from "known facts". [Why the quotes?]

BTW: How is **induction** different from **deduction**?

Simple examples:

How do you know that the sun will rise tomorrow morning?

How do you derive the formula for the sum of integers 1 .... N?

Propositional logic

What is a **proposition**?

What is a **truth value**?

Symbol T stands for truth value TRUE, and F for FALSE.

Example:

Given:

1. If it rains, the road will be wet.

2. It is raining.

We deduce (even in a dark room):

3. The road is wet.

We can represent this logic using symbols.

1. X 🡪 Y This is known as an *implication*, X *implies* Y. Here 🡪 is a *connective*.

2. X The truth of X is *asserted*

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3. Y therefore the truth of Y is *deduced*

We can say:

((X 🡪 Y) & X) 🡺 Y This is a rule of deduction. 🡺 denotes deduction.

X is the *antecedent*, and Y is the *consequence of implication.*

This is a valid rule of deductive logic.

Conversely, we have another rule of deduction.

((X 🡪 Y) & ~Y) 🡺 ~X

This second rule is the basis for *proof by contradiction*.

Use of symbols and common notation. Students should be familiar.

Truth table of *implication*:

|  |  |  |
| --- | --- | --- |
| **X** | **Y** | **X 🡪 Y** |
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Note the significance of the single F entry. It means, in everyday language that:

*It cannot happen that X is true and Y is false.*

Note that X 🡪 Y is a also a proposition, but a compound proposition, whereas X and Y are symbols for any two atomic propositions.

*Biconditional*, also known as *equivalence*: (X 🡪 Y) & (Y 🡪 X)

Rules of deduction *versus* actual deduction.

The latter is actually a proof; its steps can be listed using 🡺 or ‘therefore’ ... *et* *cetera*.

Recall that Euclid defined geometry using just a few axioms. You have seen many proofs in geometry and other branches of mathematics. All of mathematics has axiomatic basis.

[Once we introduce *probability*, we shall say, for example, that: “The probability of proposition X is being true is 0.2", or Prob(X) = 0.2.]

Thus the language of logic will prove to be useful later.

Now suppose we know that the roads are wet.

Can we conclude that it has rained?

No.

Does the statement “It has rained" become more likely to be true? In other words, does it acquire a greater *degree of plausibility*?

In other words, we believe that Prob( R | W ) > Prob( R | ~W ).

The following standard logical equivalences are useful in practice:

